Three-way Analysis of Multichannel EEG Data Using the PARAFAC and Tucker Models

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Abstract. *Changes of hidden sources of the neural electrical activity of a brain over time, as represented by a continuously recorded multichannel electroencephalogram (EEG) at the scalp, can be detected by tensor or multiway decomposition of the EEG records. In this study, the performance of i) the constrained Tucker model and ii) the parallel factor analysis (PARAFAC) models are compared on real EEG data.*

Keywords: Multichannel Electroencephalogram, Tensor Decomposition, PARAFAC, Tucker Model

Introduction

The multichannel EEG provides a useful tool for the description of the neural activity of a brain. Analysis of multichannel spatially distributed EEG information is preferred over a separate analysis of EEG signal from every single electrode.

The Tucker model [1], and its restricted version the parallel factor analysis (PARAFAC) [2, 3] are two methods developed for detection of hidden factors of multiway data, such as the multichannel EEG recorded over time.

In this study, we validate and compare both methods on EEG data recorded during the mirror therapy training of a patient after a stroke. With the aim to detect oscillatory EEG rhythms associated with the motor activity of the subject, we analysed the data using the PARAFAC model [4]. However, the resemblance of the observed spatial distribution of several in frequency not overlapped oscillatory sources motivates us to investigate a more flexible Tucker model, with the expectation of a more parsimonious representation of data.

Therefore, the aim of the article is to compare the performance of the Tucker model with PARAFAC and to choose a more compact model helping us to better interpret and represent hidden sources of the neural electrical activity of the recorded data.

Subject and Methods

Data

The subject was a 58-year-old man who had a right-hand hemiplegia due to an ischemic stroke that had occurred to him 2 years before he entered the study. The EEG recording was performed during the mirror box training when the patient tried to move both hands. Multichannel EEG signal with the sampling frequency of 512 Hz was recorded at 10 electrodes (FC3, C1, C3, C5, CP3 and FC4, C2, C4, C6, CP4) and referenced to averaged earlobes.

After careful semi-automatic artifact detection and removal step, the data were downsampled to 128 Hz and divided into 2-second-long time segments with the overlapping period of 250 ms. Then, the oscillatory spectral part of the logarithmically transformed power spectrum densities in the range of 0 - 64 Hz was extracted using the irregular resampling analysis method [5], which was applied to each electrode separately.

Using this oscillatory spectral data representation, the three-way tensor $X \in \mathbb{R}^{I\times J\times K}$ with I time segments, J electrodes, and K frequencies was constructed and zero-mean centered across the first dimension

$$x_{ijk}^{cent} = x_{ijk} - \frac{1}{I}\sum_{i=1}^{I} x_{ijk}, \qquad i = 1,\ldots,I;\;\; j = 1,\ldots,J;\;\; k = 1,\ldots,K.$$

Parallel Factor Analysis - PARAFAC

The PARAFAC model [2, 3] is a generalisation of principal component analysis (PCA) to higher dimensions. The three-way PARAFAC model provides a decomposition of the tensor $X \in \mathbb{R}^{I\times J\times K}$ into matrices $A \in \mathbb{R}^{I\times F}$, $B \in \mathbb{R}^{J\times F}$, and $C \in \mathbb{R}^{K\times F}$

$$x_{ijk} = \sum_{f=1}^{F} \lambda_f a_{if} b_{jf} c_{kf} + \eta_{ijk}, \qquad i = 1,\ldots,I;\;\; j = 1,\ldots,J;\;\; k = 1,\ldots,K,$$

by minimising the sum of squared residuals. The elements $\lambda_1,\ldots,\lambda_F$ are scaling factors, because the columns of A, B and C are assumed to have unit length. The solution is unique under very mild assumptions [6], which makes PARAFAC a powerful tool for multiway data analysis.

To allow easier and clear neurophysiological interpretation of the results, the loading matrices A (time components), B (space components) and C (frequency components) were constrained to be nonnegative. In the case of C also unimodality of its columns was considered.

Tucker model

In contrast to PARAFAC, the considered three-way Tucker model does not assume the same number of factors within each dimension. The columns of the loading matrices $\tilde{A} \in \mathbb{R}^{I\times M}, \tilde{B} \in \mathbb{R}^{J\times N}$, and $\tilde{C} \in \mathbb{R}^{K\times O}$ are mixed together by a core tensor $G \in \mathbb{R}^{M\times N\times O}$

$$x_{ijk} = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{o=1}^{O} g_{mno}\tilde{a}_{im}\tilde{b}_{jn}\tilde{c}_{ko} + \varepsilon_{ijk}, \qquad i = 1,\ldots,I;\;\; j = 1,\ldots,J;\;\; k = 1,\ldots,K.$$

The Tucker model is less restrictive than PARAFAC but at the cost of rotation freedom of the solution. However, the non-uniqueness of the solution can be solved by restrictions to the loading matrices.

Similarly to PARAFAC, we considered the non-negativity constraints for $\tilde{A}$, $\tilde{B}$ and the non-negativity and unimodality constraints for columns of $\tilde{C}$. The core tensor G was set to be non-negative and to have diagonal lateral slices. In other words, a frequency vector (a column of $\tilde{C}$) is connected with only one time vector (a column of $\tilde{A}$), but it can be related to different space vectors (columns of $\tilde{B}$). Consequently, $M = O$ and $g_{mno} \neq 0$ only if $m = o$; $m, o = 1,\ldots,M$.

These constraints have made the Tucker model to be "equivalent" with PARAFAC

$$x_{ijk} = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{o=1}^{O} g_{mno}\tilde{a}_{im}\tilde{b}_{jn}\tilde{c}_{ko} + \varepsilon_{ijk} = \sum_{m=1}^{M} \tilde{a}_{im}\tilde{c}_{km}\left(\sum_{n=1}^{N} g_{mnm}\tilde{b}_{jn}\right) + \varepsilon_{ijk} = \sum_{m=1}^{M} \tilde{a}_{im}\tilde{c}_{km}\tilde{b}^{\star}_{jm} + \varepsilon_{ijk}.$$

Due to the uniqueness of the PARAFAC solution, the estimates of $\tilde{A}$ and $\tilde{C}$ are also unique. Moreover, the space components in PARAFAC are a linear combination of space components estimated by the Tucker model.

Setting the number of components in PARAFAC and Tucker model

The number of components $F = 6$ in PARAFAC and $M = 6$, $N = 2$ in the Tucker model were chosen to balance minimisation of the mean squared error (MSE) and maximisation of the explained variance and the core consistency diagnostics (CorConDiag) [7].

CorConDiag $\in (-\infty, 1]$ represents the appropriateness of PARAFAC or the constrained Tucker model in comparison to an unconstrained Tucker model with the same factors. Values close to 1 indicate that the constraints in the model are appropriate.

Results and Discussion

Both, the PARAFAC and Tucker models performed similarly when analysing EEG records from all 11 days. This was true considering the MSE ($\approx 4.73 \times 10^8$) measure, the proportion of explained variance ($\approx 6.4\%$) or visual inspection of the time and frequency components (Fig. 1). However, the Tucker model provided approximately the same results with a lower number of factors. Moreover, the CorConDiag values of the Tucker model were at least two–times higher than in PARAFAC (Table 1), favouring the Tucker model.

Next, estimated time, space and frequency components from the 4^{th} day are analysed. Results for other days were similar.

The space components estimated by PARAFAC visually follow two different profiles (Fig. 2). These profiles represent either the right (components 5, 6) or the left hemisphere (components 3, 4). Component 2 (8 Hz) is located predominately in the right hemisphere, but in comparison to components 5 or 6, it shows higher weights also for electrodes on the left part of the scalp. Component 1 represents the neural activity of 6 Hz symmetrically located at both hemispheres.

The first space component in the Tucker model (Fig. 3) represents the neural activity in the left hemisphere and is related mainly with the frequency components 3 and 4, see higher scores for these components in the first lateral slice of the tensor G (values 2.11×10^3 and 2.61×10^3). On the other hand, the frequency components 5 and 6 achieved higher scores (2.20×10^3 and 1.97×10^3) for the second space component that represents the right hemisphere (Fig. 3). The frequency components 1 and 2 are related to both space components which is consistent with the results obtained by PARAFAC.

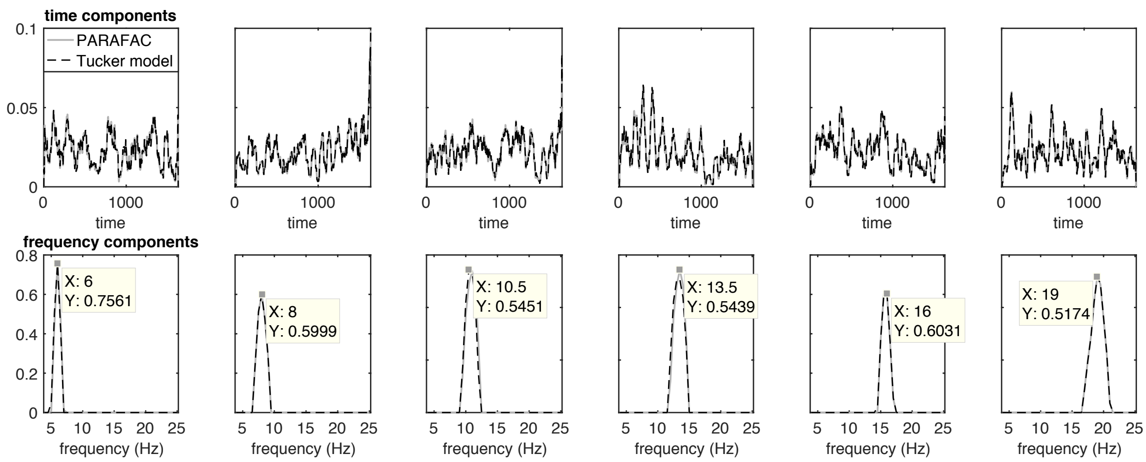


Fig. 1: The PARAFAC and Tucker decomposition of the 4^{th} day EEG data. The top row represents time components, the corresponding frequency components are depicted in the bottom row.

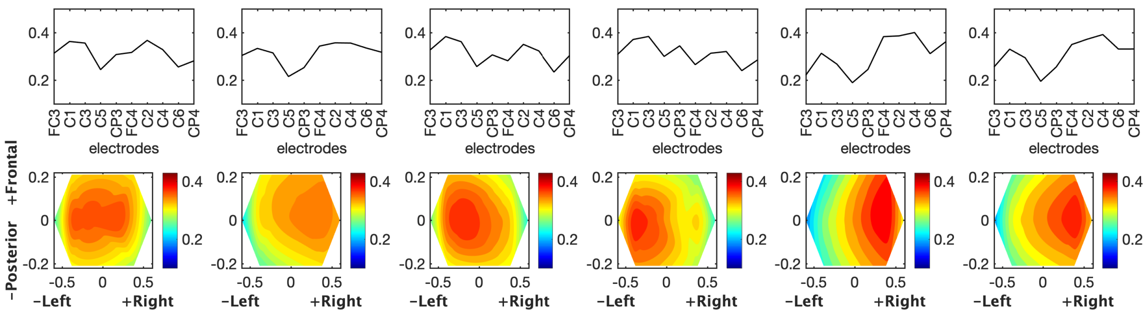
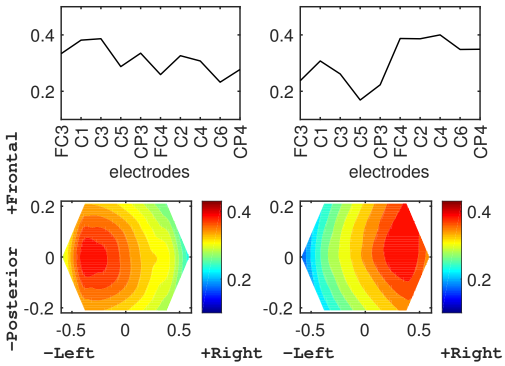


Fig. 2: The space components of the 4^{th} day EEG data estimated by PARAFAC (top row). The corresponding topographical maps are depicted in the bottom row.



$$G_{.1.} = \begin{pmatrix} \mathbf{1.62} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.08 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{2.11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{2.61} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.62 \end{pmatrix} \times 10^3$$

$$G_{.2.} = \begin{pmatrix} 0.79 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1.80} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.48 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.05 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{2.20} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{1.97} \end{pmatrix} \times 10^3$$

Fig. 3: *Left:* The space components of the 4^{th} day EEG data estimated by the Tucker model (top row) and the corresponding topographical maps (bottom row). *Right:* The lateral slices of the core tensor G.

Table 1: The core consistency diagnostic (CorConDiag) of PARAFAC and the Tucker model.

Day	1	2	3	4	5	6	7	8	9	10	11
PARAFAC	0.32	0.17	0.13	0.22	0.23	0.17	0.04	0.19	0.20	0.12	0.18
Tucker	0.68	0.79	0.59	0.73	0.82	0.67	0.69	0.72	0.75	0.77	0.77

Conclusion

For all 11 investigated days, the PARAFAC and the restricted version of the Tucker model produced similar decompositions of the oscillatory part of the EEG power spectra. This was true considering either the MSE, the proportion of explained variance, visual inspection or location of neural activity on different frequencies in the left or right hemisphere. However, higher CorConDiag values and a lower number of components needed to describe the data variability indicate that the Tucker model is preferred. Further validation of the result will be studied using higher density EEG recordings with channels distributed over the whole scalp.

Acknowledgements

This research was supported by the Slovak Research and Development Agency (grant APVV-16-0202) and by the VEGA grant 2/0081/19.

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