Three-way Analysis of Multichannel EEG Data Using the PARAFAC and Tucker Models

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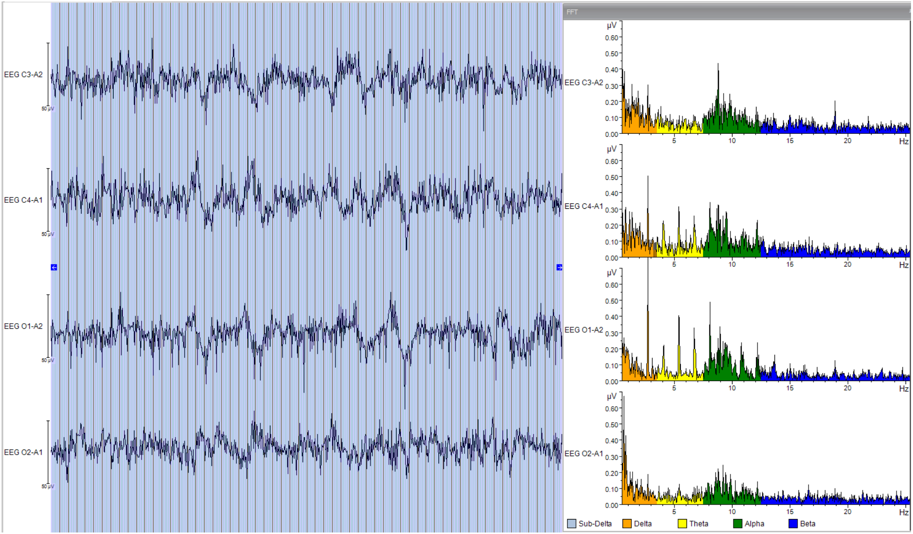
Electroencephalogram – EEG



Mirror-box therapy

- $\theta \in [4, 6.5]$ Hz
- $\mu \in [7, 8.5]$ Hz
- $\alpha \in [9, 11.5]$ Hz
- $SMR \in [12, 14.5]$ Hz
- $\beta \in [15, 20]$ Hz

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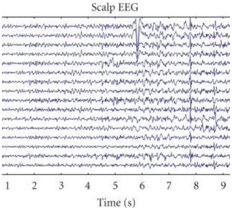
1. EEG recording



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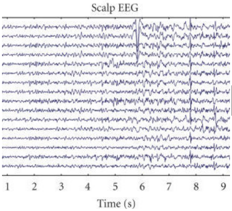
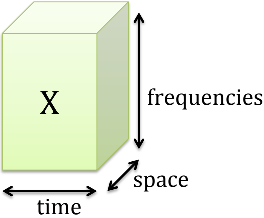
1. EEG recording **2. Spectral analysis**





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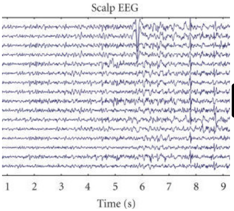
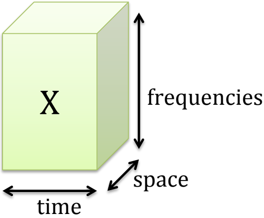
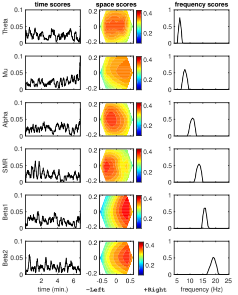


2. Spectral analysis

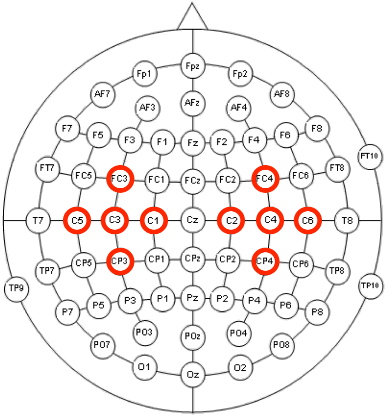


3. Atomic decomposition



1. EEG recording and preprocessing

- 58–year–old men
- ischemic stroke 2 years before the study; right–hand hemiplegia
- 11 days/sessions of the mirror–box therapy

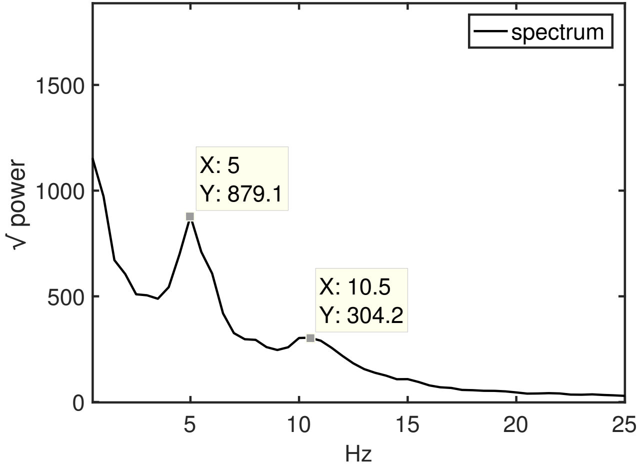


http://www.fieldtriptoolbox.org/faq/capmapping/

- **EEG preprocessing:**
  - artefact detection
  - 2–second–long time segments, overlapping period 250ms

2. Spectral analysis

- **Irregular–Resampling Auto–Spectral Analysis** (IRASA)
  [Wen and Liu, 2016]
- separation of fractal and oscillatory components in the power spectrum



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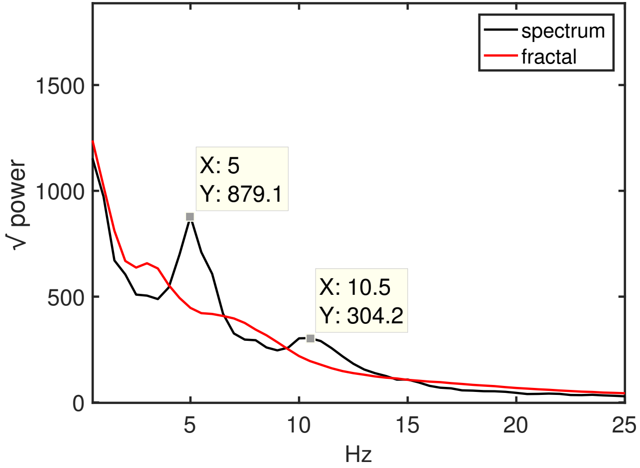

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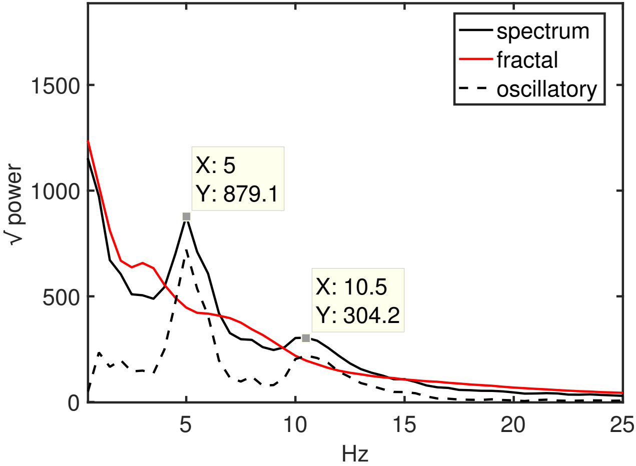

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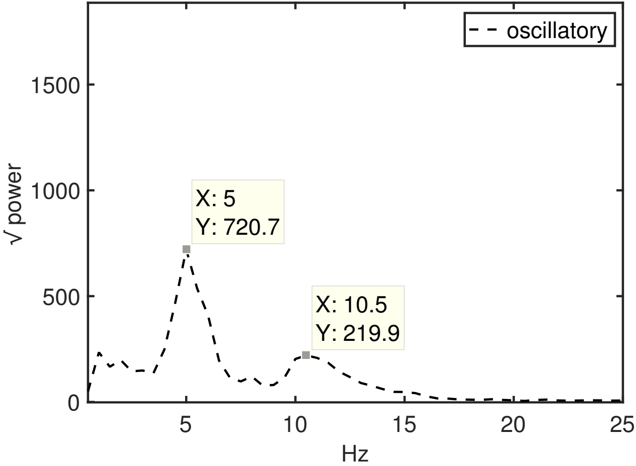
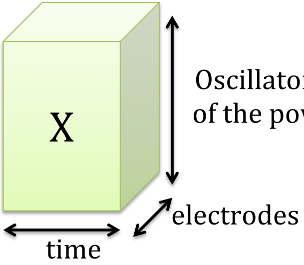


3. Atomic decomposition

- **goal:** to detect hidden sources of neural activity

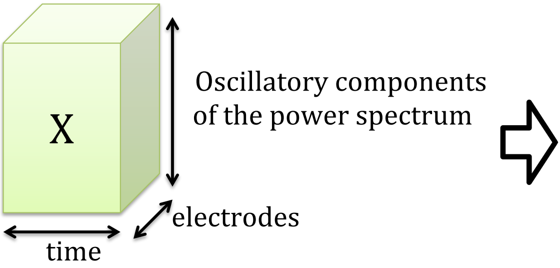


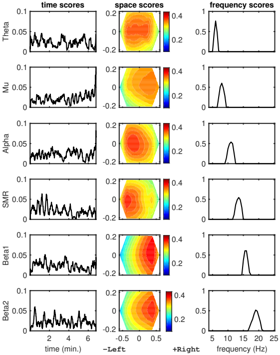
3. Atomic decomposition

- **goal:** to detect hidden sources of neural activity

⇒ to detect "atoms", which are represented by their

  - time scores – time periods, when the atom was active
  - space scores – location of the "atom" on the scalp
  - frequency scores – frequency typical for the "atom"





3. Atomic decomposition – methods

- **Parallel Factor Analysis**

  [Harshman, 1970, Carroll and Chang, 1970]

- **Tucker model**

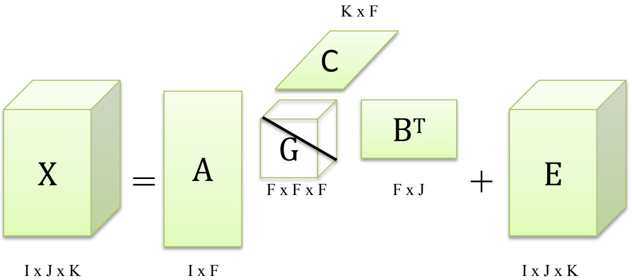
  [Tucker, 1966, Kroonenberg, 1983]

PARAllel FACtor Analysis (PARAFAC)

$X \in \mathbb{R}^{\mathbb{I}\times\mathbb{J}\times\mathbb{K}}$:

$$X_{ijk} = \sum_{f=1}^{F} g_{fff}\, a_{if}\, b_{jf}\, c_{kf} + e_{ijk},$$

$$\|a_f\| = \|b_f\| = \|c_f\| = 1$$



→ **restrictions:** nonnegativity; unimodality of columns in C

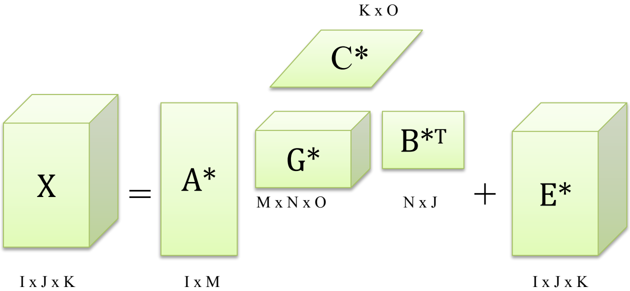
→ **uniqueness:** unique solution under very mild conditions

Tucker model

$X \in \mathbb{R}^{\mathbb{I}\times\mathbb{J}\times\mathbb{K}}$:

$$X_{ijk} = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{o=1}^{O} g^{\star}_{mno} a^{\star}_{im} b^{\star}_{jn} c^{\star}_{ko} + e^{\star}_{ijk},$$

$$\|a_m\| = \|b_n\| = \|c_o\| = 1$$



→ **restrictions:** nonnegativity; unimodality of columns in C

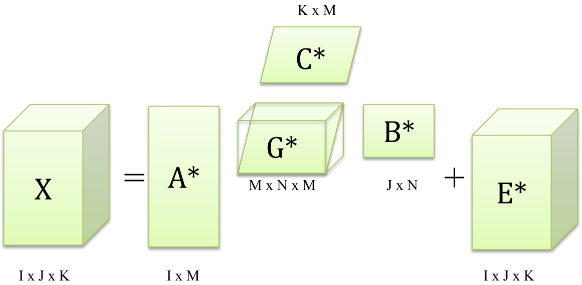
→ **uniqueness:** the solution is **not** unique → rotation freedom

Tucker model - version with restricted $G^\star$

$X \in \mathbb{R}^{\mathbb{I}\times\mathbb{J}\times\mathbb{K}}$:

$$x_{ijk} = \sum_{m=1}^{M}\sum_{n=1}^{N} g^\star_{mnm} a^\star_{im} b^\star_{jn} c^\star_{km} + e^\star_{ijk},$$

$$\|a_m\| = \|b_n\| = \|c_m\| = 1$$



→ **restrictions:** nonnegativity; unimodality of columns in C; diagonal lateral slices of $G^\star$

→ **uniqueness:** $A^\star, C^\star$ are unique; $B^\star$ can be rotated

Model comparison – criteria

- **visual and physiological interpretation**
- **proportion of variance explained**

$$VarExpl = 100 \times \left(1 - \frac{\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \left(X_{ijk} - \widehat{X}_{ijk} \right)^2}{\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} X_{ijk}^2} \right)$$

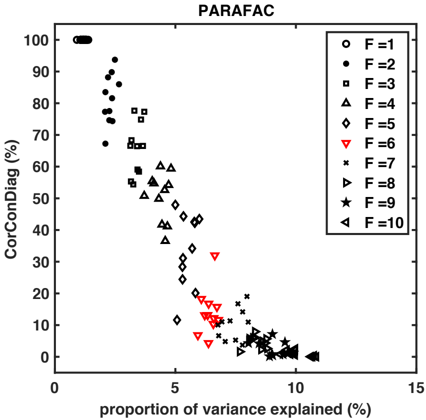
- **core consistency diagnostics** [Bro and Kiers, 2003]

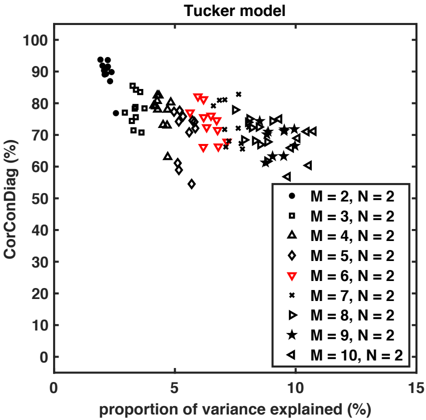
$$CorConDiag = 100 \times \left(1 - \frac{\sum_{m=1}^{F} \sum_{n=1}^{F} \sum_{o=1}^{F} \left(g_{mno} - g_{mno}^{\star} \right)^2}{{g_{mno}^{\star}}^2} \right) \in (-\infty, 100]$$

1. estimate A, B, C and G in PARAFAC/restricted Tucker model
2. estimate $G^{\star}$ in unrestricted Tucker model with A, B, C from step 1

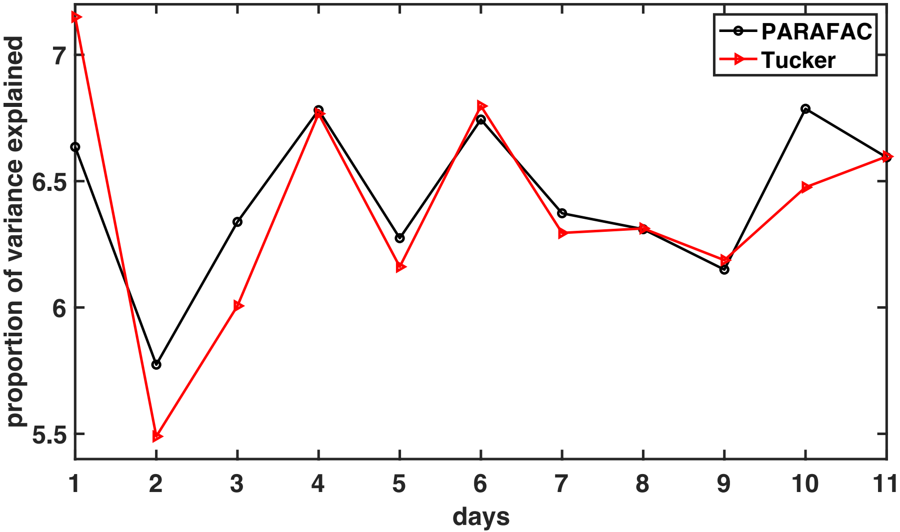
Number of components

- **PARAFAC:** F = 6
- **Tucker model:** M = 6, N = 2

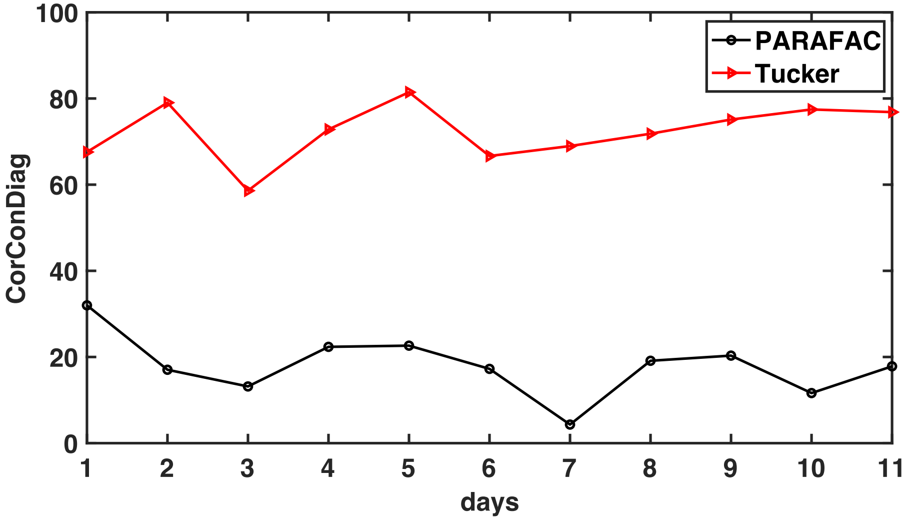




Results

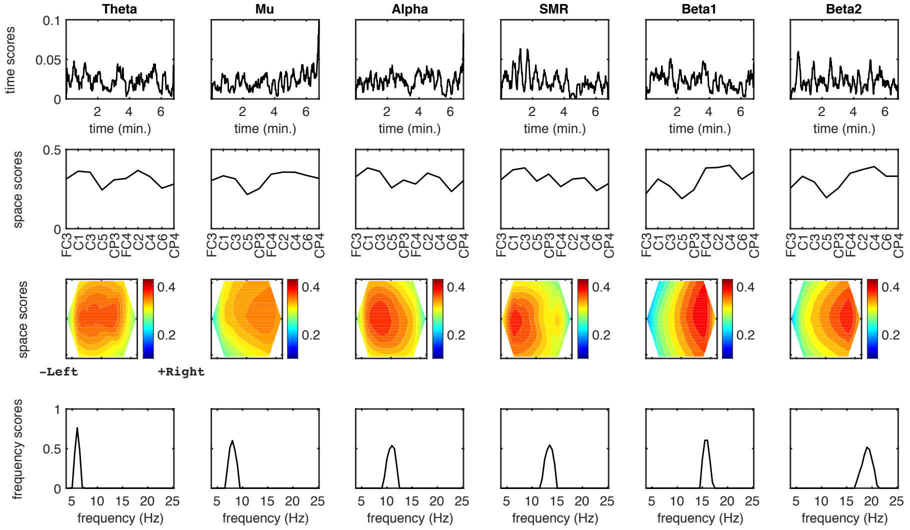

PARAFAC
Tucker
proportion of variance explained
7
6.5
6
5.5
1
2
3
4
5
6
7
8
9
10
11
days

Results

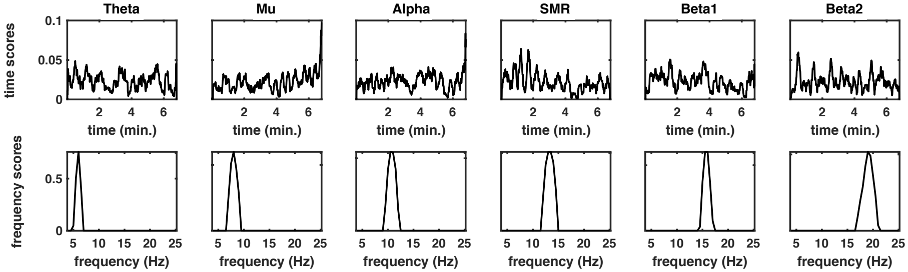


PARAFAC
Tucker
CorConDiag
days
100
80
60
40
20
0
1
2
3
4
5
6
7
8
9
10
11

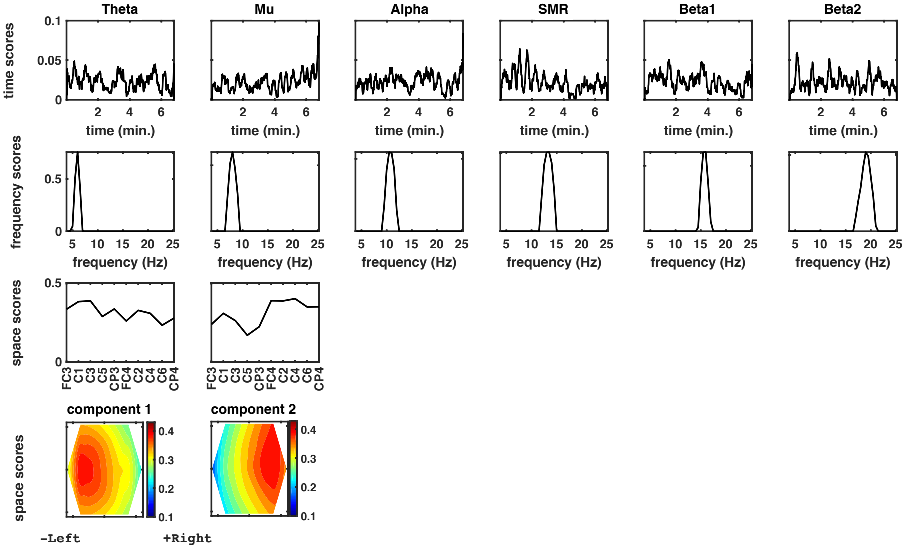
PARAFAC – 4^{th} day



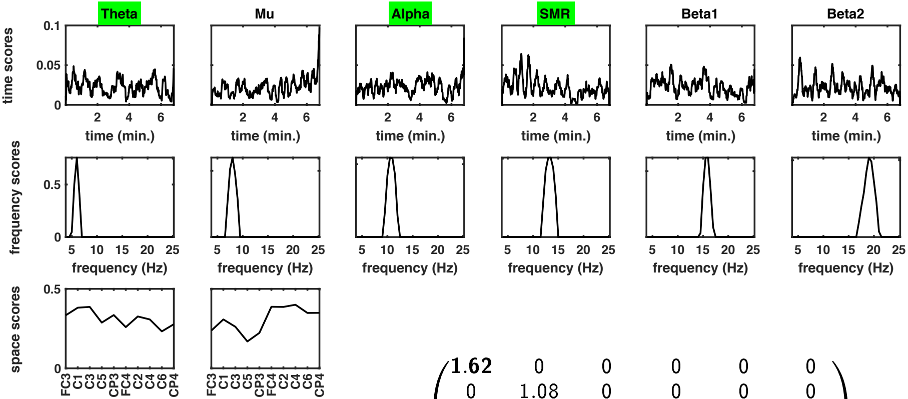
Tucker model – 4^{th} day

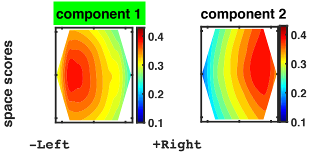


Tucker model – 4^{th} day



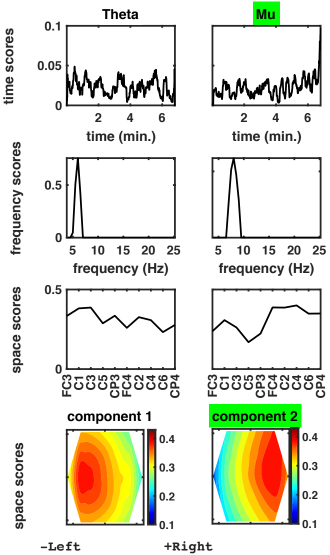
Tucker model – 4^{th} day





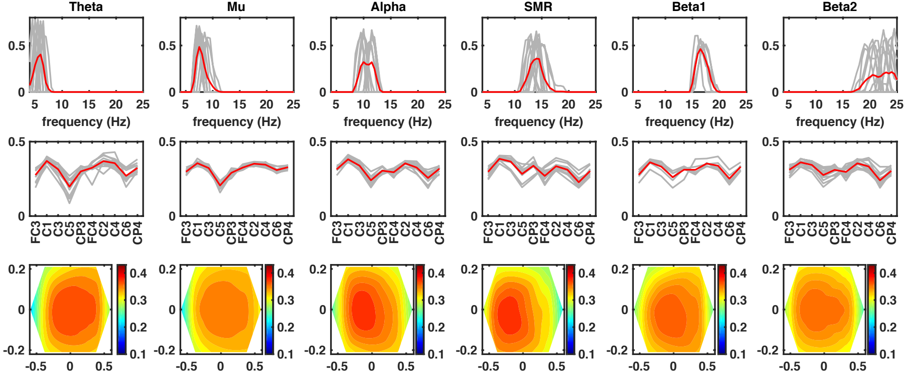
$$G_{1.} = \begin{pmatrix} \mathbf{1.62} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.08 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{2.11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{2.61} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.62 \end{pmatrix} \times 10^3$$

Tucker model – 4^{th} day

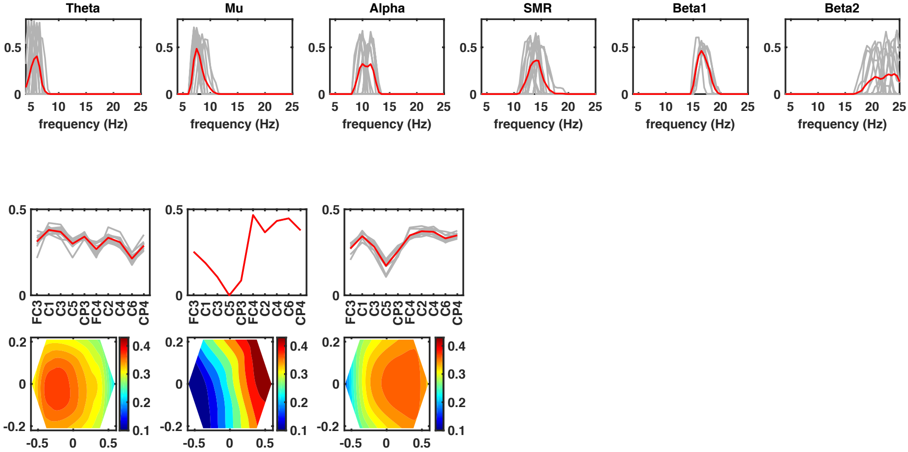


$$G_{2.} = \begin{pmatrix} 0.79 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1.80} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.48 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.05 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{2.20} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{1.97} \end{pmatrix} \times 10^3$$

PARAFAC – average of 11 days



Tucker model – average of 11 days



Conclusion

- **comparison of PARAFAC and the Tucker model**
  - successful extraction of the sensori–motor oscillatory rhythms
  - meaningful neurophysiological interpretation of the results
  - the models yielded similar results in terms of
    - mean squared error, proportion of variance explained
    - time and frequency components
  - **the Tucker model overcomes PARAFAC** in
    - the CorConDiag values
    - the lower number of frequency components needed to describe the same amount of the data variability
  - further validation of the result by using higher density EEG recordings

Literature

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